Interactions, 1

```
library(ISLR)
Auto$foreign=ifelse(Auto$origin==1,0,1)
fit.linear <- lm(mpg ~ horsepower + weight + foreign + year, data=Auto)
summary(fit.linear)
```

```
Call:
lm(formula = mpg ~ horsepower + weight + foreign + year, data = Auto)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.4008 -2.1335 -0.0628  1.8284 13.6148 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.758e+01  4.143e+00  -4.242 2.77e-05 ***
horsepower  -7.541e-03  9.192e-03  -0.820    0.413    
weight      -5.608e-03  4.338e-04 -12.928  < 2e-16 ***
foreign      2.116e+00  4.370e-01   4.841 1.87e-06 ***
year         7.596e-01  5.073e-02  14.975  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.335 on 387 degrees of freedom
Multiple R-squared:  0.8193,    Adjusted R-squared:  0.8174 
F-statistic: 438.6 on 4 and 387 DF,  p-value: < 2.2e-16
```

```
fit.inter <- lm(mpg ~ horsepower + weight + foreign + year +
foreign*weight, data=Auto)
```

```
summary(fit.inter)
```

```
Call:
lm(formula = mpg ~ horsepower + weight + foreign + year + foreign *
    weight, data = Auto)

Residuals:
    Min      1Q  Median      3Q     Max
-9.7988 -2.0742 -0.1083  1.6348 12.8584

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)    -2.179e+01  4.102e+00  -5.313 1.83e-07 ***
horsepower     -1.190e-02  8.953e-03  -1.330    0.184
weight         -4.876e-03  4.446e-04 -10.969  < 2e-16 ***
foreign         1.091e+01  1.784e+00   6.114 2.38e-09 ***
year            7.896e-01  4.953e-02  15.942  < 2e-16 ***
weight:foreign -3.544e-03  6.987e-04  -5.073 6.09e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.234 on 386 degrees of freedom
Multiple R-squared:  0.8306,    Adjusted R-squared:  0.8284
F-statistic: 378.4 on 5 and 386 DF,  p-value: < 2.2e-16
```

Interactions, 2

```
fit.inter.2 <- lm(mpg ~ horsepower + weight + foreign + year +
foreign*weight + foreign*year, data=Auto)
summary(fit.inter.2)
```

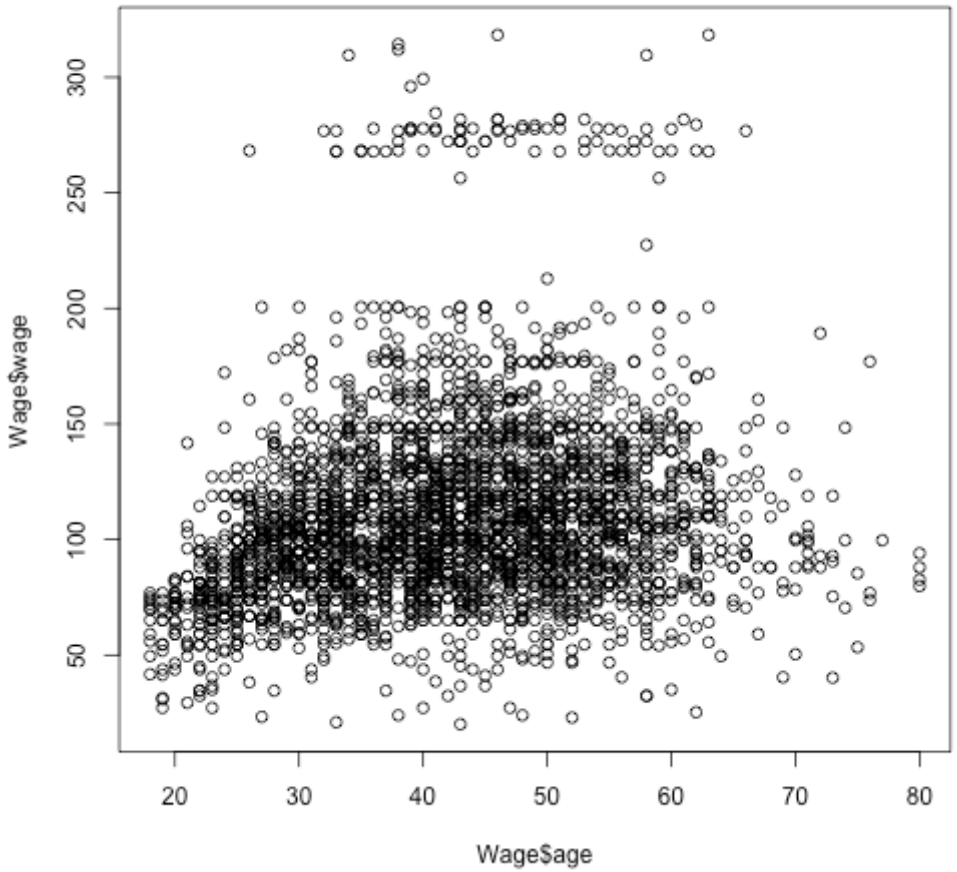
```
Call:
lm(formula = mpg ~ horsepower + weight + foreign + year + foreign *
    weight + foreign * year, data = Auto)

Residuals:
    Min      1Q  Median      3Q     Max
-8.8995 -1.9335 -0.0871  1.5693 12.0042

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)    -7.263e+00  5.182e+00  -1.402  0.16184
horsepower     -1.531e-02  8.780e-03  -1.743  0.08205 .
weight         -5.067e-03  4.364e-04 -11.611  < 2e-16 ***
foreign        -2.163e+01  7.562e+00  -2.861  0.00446 **
year            6.113e-01  6.299e-02   9.704  < 2e-16 ***
weight:foreign -3.346e-03  6.839e-04  -4.892 1.47e-06 ***
foreign:year    4.167e-01  9.424e-02   4.422 1.28e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.158 on 385 degrees of freedom
Multiple R-squared:  0.8388,    Adjusted R-squared:  0.8362
F-statistic: 333.8 on 6 and 385 DF,  p-value: < 2.2e-16
```

```r
anova(fit.linear, fit.inter, fit.inter.2)
```

```
Analysis of Variance Table

Model 1: mpg ~ horsepower + weight + foreign + year
Model 2: mpg ~ horsepower + weight + foreign + year + foreign * weight
Model 3: mpg ~ horsepower + weight + foreign + year + foreign * weight +
    foreign * year
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    387 4304.9
2    386 4035.8  1    269.10 26.975 3.348e-07 ***
3    385 3840.8  1    195.05 19.552 1.275e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Polynomials, 1

```
library(ISLR)
plot(Wage$age, Wage$wage)
```



```r
fit.poly1=lm(wage~age,data=Wage)
summary(fit.poly1)
```

```
Call:
lm(formula = wage ~ age, data = Wage)

Residuals:
     Min       1Q   Median       3Q      Max
-100.265  -25.115   -6.063   16.601  205.748

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 81.70474    2.84624   28.71   <2e-16 ***
age          0.70728    0.06475   10.92   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 40.93 on 2998 degrees of freedom
Multiple R-squared:  0.03827,   Adjusted R-squared:  0.03795
F-statistic: 119.3 on 1 and 2998 DF,  p-value: < 2.2e-16
```

```r
fit.poly2=lm(wage~poly(age,2),data=Wage)
summary(fit.poly2)
```

```
Call:
lm(formula = wage ~ poly(age, 2), data = Wage)

Residuals:
    Min      1Q  Median      3Q     Max
-99.126 -24.309  -5.017  15.494 205.621

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)    111.7036     0.7302  152.99   <2e-16 ***
poly(age, 2)1  447.0679    39.9926   11.18   <2e-16 ***
poly(age, 2)2 -478.3158    39.9926  -11.96   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.99 on 2997 degrees of freedom
Multiple R-squared:  0.08209,   Adjusted R-squared:  0.08147
F-statistic:   134 on 2 and 2997 DF,  p-value: < 2.2e-16
```

```
fit.poly4=lm(wage~poly(age,4),data=Wage)
summary(fit.poly4)
```

```

Call:
lm(formula = wage ~ poly(age, 4), data = Wage)

Residuals:
    Min      1Q  Median      3Q     Max
-98.707 -24.626  -4.993  15.217 203.693

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    111.7036     0.7287 153.283  < 2e-16 ***
poly(age, 4)1  447.0679    39.9148  11.201  < 2e-16 ***
poly(age, 4)2 -478.3158    39.9148 -11.983  < 2e-16 ***
poly(age, 4)3  125.5217    39.9148   3.145  0.00168 **
poly(age, 4)4  -77.9112    39.9148  -1.952  0.05104 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.91 on 2995 degrees of freedom
Multiple R-squared:  0.08626,   Adjusted R-squared:  0.08504
F-statistic: 70.69 on 4 and 2995 DF,  p-value: < 2.2e-16
```

```r
anova(fit.poly1, fit.poly2, fit.poly4)
```

```
Analysis of Variance Table

Model 1: wage ~ age
Model 2: wage ~ poly(age, 2)
Model 3: wage ~ poly(age, 4)
  Res.Df     RSS Df Sum of Sq        F    Pr(>F)
1   2998 5022216
2   2997 4793430  1    228786 143.6025 < 2.2e-16 ***
3   2995 4771604  2     21826   6.8497  0.001076 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Polynomials, 2

```
fit.poly4=lm(wage~poly(age,4),data=Wage)
summary(fit.poly4)
```

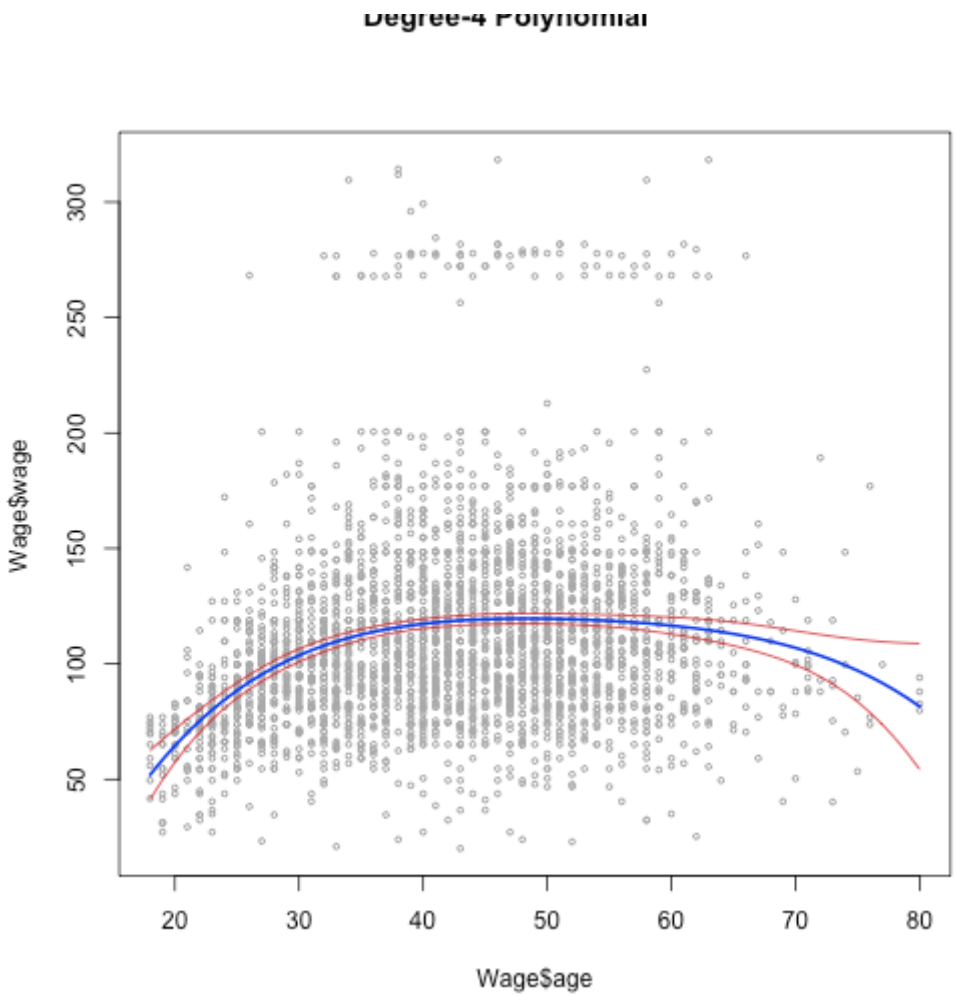
```
Call:
lm(formula = wage ~ poly(age, 4), data = Wage)

Residuals:
    Min      1Q  Median      3Q     Max 
-98.707 -24.626  -4.993  15.217 203.693 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)    111.7036     0.7287 153.283  < 2e-16 ***
poly(age, 4)1  447.0679    39.9148  11.201  < 2e-16 ***
poly(age, 4)2 -478.3158    39.9148 -11.983  < 2e-16 ***
poly(age, 4)3  125.5217    39.9148   3.145  0.00168 ** 
poly(age, 4)4  -77.9112    39.9148  -1.952  0.05104 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.91 on 2995 degrees of freedom
Multiple R-squared:  0.08626,   Adjusted R-squared:  0.08504 
F-statistic: 70.69 on 4 and 2995 DF,  p-value: < 2.2e-16
```

```
fit.poly4.I=lm(wage~age+I(age^2)+I(age^3)+I(age^4),data=Wage)
summary(fit.poly4.I)
```

```
Call:
lm(formula = wage ~ age + I(age^2) + I(age^3) + I(age^4), data = Wage)

Residuals:
    Min      1Q  Median      3Q     Max
-98.707 -24.626  -4.993  15.217 203.693

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.842e+02  6.004e+01  -3.067 0.002180 **
age          2.125e+01  5.887e+00   3.609 0.000312 ***
I(age^2)    -5.639e-01  2.061e-01  -2.736 0.006261 **
I(age^3)     6.811e-03  3.066e-03   2.221 0.026398 *
I(age^4)    -3.204e-05  1.641e-05  -1.952 0.051039 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.91 on 2995 degrees of freedom
Multiple R-squared:  0.08626,   Adjusted R-squared:  0.08504
F-statistic: 70.69 on 4 and 2995 DF,  p-value: < 2.2e-16
```

Polynomials, 3

```
agelims=range(Wage$age)
plot(Wage$age,Wage$wage,xlim=agelims,cex=.5,col="darkgrey")
title("Degree-4 Polynomial",outer=T)

age.grid=seq(from=agelims[1],to=agelims[2])
preds=predict(fit.poly4, newdata=list(age=age.grid), se=TRUE)

# Plot estimate + confidence bounds
lines(age.grid,preds$fit,lwd=2, col="blue")
lines(age.grid,preds$fit-2*preds$se.fit,lwd=1, col="red")
lines(age.grid,preds$fit+2*preds$se.fit,lwd=1, col="red")
```


Degree-4 Polynomial
Wage$wage
50
100
150
200
250
300
20
30
40
50
60
70
80
Wage$age

Step Functions

```
plot(Wage$age,Wage$wage,xlim=agelims,cex=.5,col="darkgrey")
title("Step Regression of Wage on Age",outer=T)

# Fit model
fit.step=lm(wage~cut(age,4),data=Wage)
summary(fit.step) # Take a look
```

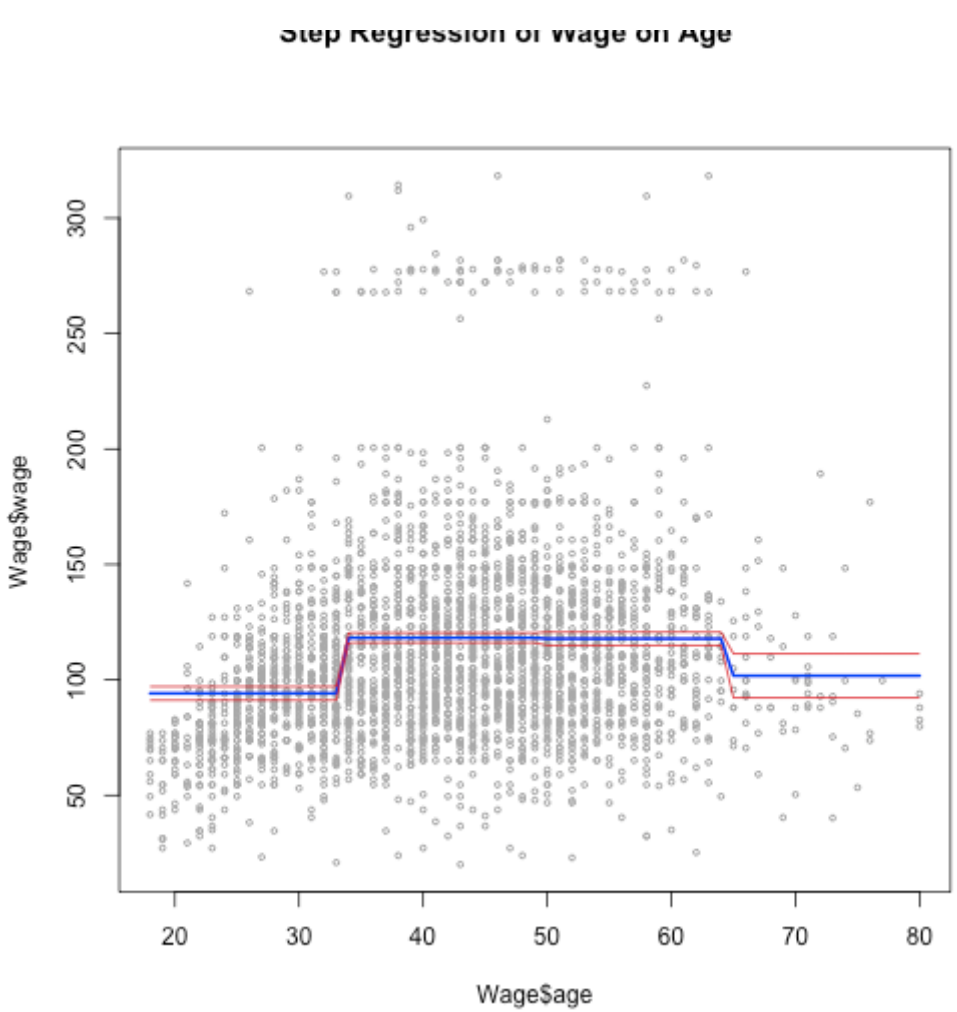
```
Call:
lm(formula = wage ~ cut(age, 4), data = Wage)

Residuals:
    Min      1Q  Median      3Q     Max
-98.126 -24.803  -6.177  16.493 200.519

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)               94.158      1.476  63.790   <2e-16 ***
cut(age, 4)(33.5,49]      24.053      1.829  13.148   <2e-16 ***
cut(age, 4)(49,64.5]      23.665      2.068  11.443   <2e-16 ***
cut(age, 4)(64.5,80.1]     7.641      4.987   1.532    0.126
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 40.42 on 2996 degrees of freedom
Multiple R-squared:  0.0625,    Adjusted R-squared:  0.06156
F-statistic: 66.58 on 3 and 2996 DF,  p-value: < 2.2e-16
```

```
# Find step function values
preds=predict(fit.step,newdata=list(age=age.grid),se=T)
preds # Take a look
```

```
$fit
        1         2         3         4         5         6         7
 94.15839  94.15839  94.15839  94.15839  94.15839  94.15839  94.15839
        8         9        10        11        12        13        14
 94.15839  94.15839  94.15839  94.15839  94.15839  94.15839  94.15839
       15        16        17        18        19        20        21
 94.15839  94.15839 118.21188 118.21188 118.21188 118.21188 118.21188
       22        23        24        25        26        27        28
118.21188 118.21188 118.21188 118.21188 118.21188 118.21188 118.21188
       29        30        31        32        33        34        35
118.21188 118.21188 118.21188 118.21188 117.82295 117.82295 117.82295
       36        37        38        39        40        41        42
117.82295 117.82295 117.82295 117.82295 117.82295 117.82295 117.82295
       43        44        45        46        47        48        49
117.82295 117.82295 117.82295 117.82295 117.82295 101.79898 101.79898
       50        51        52        53        54        55        56
101.79898 101.79898 101.79898 101.79898 101.79898 101.79898 101.79898
       57        58        59        60        61        62        63
101.79898 101.79898 101.79898 101.79898 101.79898 101.79898 101.79898

$se.fit
       1        2        3        4        5        6        7        8
1.476069 1.476069 1.476069 1.476069 1.476069 1.476069 1.476069 1.476069
       9       10       11       12       13       14       15       16
1.476069 1.476069 1.476069 1.476069 1.476069 1.476069 1.476069 1.476069
      17       18       19       20       21       22       23       24
1.080758 1.080758 1.080758 1.080758 1.080758 1.080758 1.080758 1.080758
      25       26       27       28       29       30       31       32
1.080758 1.080758 1.080758 1.080758 1.080758 1.080758 1.080758 1.080758
      33       34       35       36       37       38       39       40
1.448333 1.448333 1.448333 1.448333 1.448333 1.448333 1.448333 1.448333
      41       42       43       44       45       46       47       48
1.448333 1.448333 1.448333 1.448333 1.448333 1.448333 1.448333 4.763992
```

```
      49       50       51       52       53       54       55       56
4.763992 4.763992 4.763992 4.763992 4.763992 4.763992 4.763992 4.763992
      57       58       59       60       61       62       63
4.763992 4.763992 4.763992 4.763992 4.763992 4.763992 4.763992

$df
[1] 2996

$residual.scale
[1] 40.42381
```

```r
# Plot estimate + confidence bounds
lines(age.grid,preds$fit,lwd=2,col="blue")
lines(age.grid,preds$fit-2*preds$se.fit,lwd=1,col="red")
lines(age.grid,preds$fit+2*preds$se.fit,lwd=1,col="red")
```


Step Regression of Wage on Age
Wage$wage
50
100
150
200
250
300
20
30
40
50
60
70
80
Wage$age

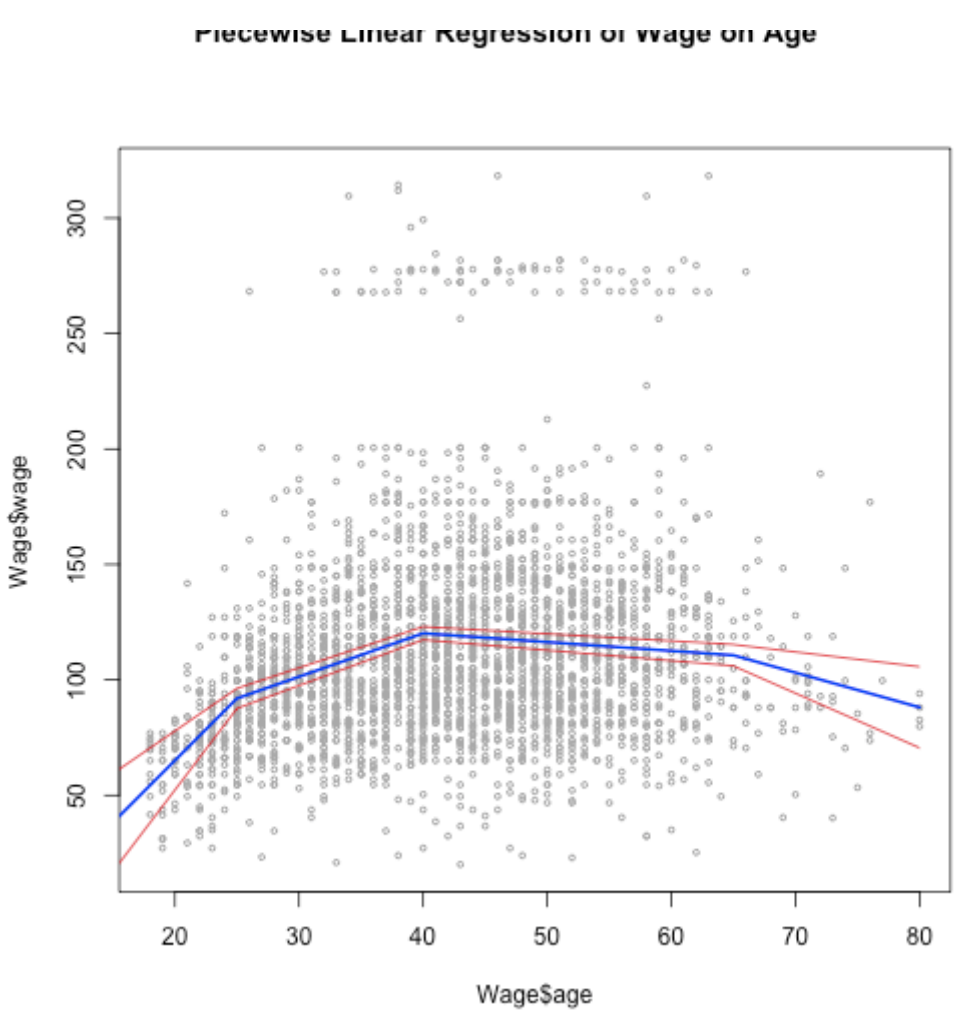
Piecewise Linear

```r
plot(Wage$age,Wage$wage,xlim=agelims,cex=.5,col="darkgrey")
title("Piecewise Linear Regression of Wage on Age",outer=T)

# Fit model
fit.piecewise=lm(wage~age+
                       I((age-25)*(age>25))+
                       I((age-40)*(age>40))+
                       I((age-60)*(age>60)),
                       data=Wage)
summary(fit.piecewise)
```

```
Call:
lm(formula = wage ~ age + I((age - 25) * (age > 25)) + I((age -
    40) * (age > 40)) + I((age - 60) * (age > 60)), data = Wage)

Residuals:
    Min      1Q  Median      3Q     Max
-99.795 -24.686  -4.856  15.344 204.671

Coefficients:
                            Estimate Std. Error t value Pr(>|t|)
(Intercept)                -42.4688    23.2695  -1.825  0.06809 .
age                          5.3779     0.9739   5.522 3.64e-08 ***
I((age - 25) * (age > 25))  -3.4977     1.0800  -3.239  0.00121 **
I((age - 40) * (age > 40))  -1.9800     0.2970  -6.667 3.10e-11 ***
I((age - 60) * (age > 60))  -1.4041     0.5424  -2.589  0.00968 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

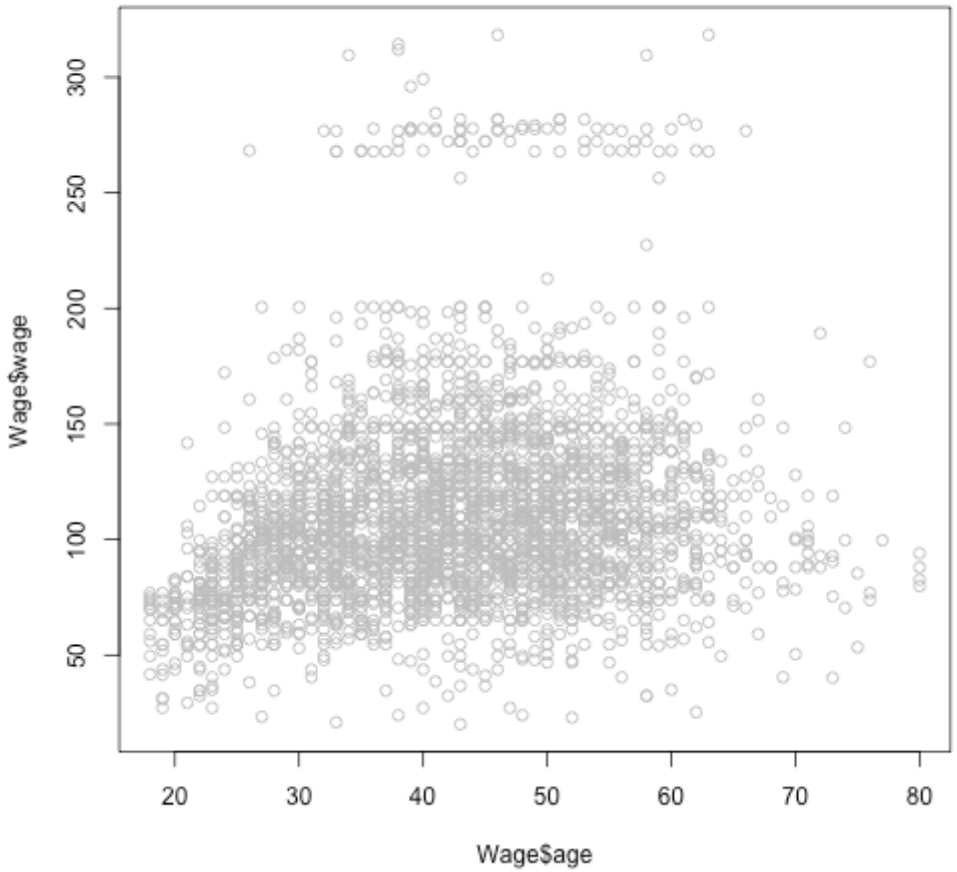
```
Residual standard error: 39.91 on 2995 degrees of freedom
Multiple R-squared:  0.08665,   Adjusted R-squared:  0.08543
F-statistic: 71.03 on 4 and 2995 DF,  p-value: < 2.2e-16
```

```r
# Find piecewise function values
age.knots=c(0,25,40,65,80)
preds=predict(fit.piecewise,list(age=age.knots), se=T)

# Plot estimate + confidence bounds
lines(age.knots,preds$fit,col="blue",lwd=2)
lines(age.knots,preds$fit-2*preds$se.fit,col="red",lwd=1)
lines(age.knots,preds$fit+2*preds$se.fit,col="red",lwd=1)
```


Piecewise Linear Regression of Wage on Age
Wage$wage
50
100
150
200
250
300
20
30
40
50
60
70
80
Wage$age

Spline Functions 1

```
library(splines)
plot(Wage$age, Wage$wage,col="gray")
```



```r
# Fit model
# Here we specify knots at age = 25, 40, 60
fit.x1=lm(wage~bs(age,knots=c(25,40,60), degree=1),data=Wage)
# Or, evently spaced knots
# fit.x1.df=lm(wage~bs(age,df=4,degree=1),data=Wage)
summary(fit.x1)
```

```
Call:
lm(formula = wage ~ bs(age, knots = c(25, 40, 60), degree = 1),
    data = Wage)

Residuals:
    Min      1Q  Median      3Q     Max
-99.795 -24.686  -4.856  15.344 204.671

Coefficients:
                                            Estimate Std. Error t value
(Intercept)                                   54.333      5.957   9.120
bs(age, knots = c(25, 40, 60), degree = 1)1   37.645      6.817   5.522
bs(age, knots = c(25, 40, 60), degree = 1)2   65.847      6.019  10.940
bs(age, knots = c(25, 40, 60), degree = 1)3   63.850      6.319  10.104
bs(age, knots = c(25, 40, 60), degree = 1)4   33.772     10.580   3.192
                                            Pr(>|t|)
(Intercept)                                  < 2e-16 ***
bs(age, knots = c(25, 40, 60), degree = 1)1 3.64e-08 ***
bs(age, knots = c(25, 40, 60), degree = 1)2  < 2e-16 ***
bs(age, knots = c(25, 40, 60), degree = 1)3  < 2e-16 ***
bs(age, knots = c(25, 40, 60), degree = 1)4  0.00143 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 39.91 on 2995 degrees of freedom
Multiple R-squared:  0.08665,   Adjusted R-squared:  0.08543
F-statistic: 71.03 on 4 and 2995 DF,  p-value: < 2.2e-16
```

Spline Functions 2

```
# plot(Wage$age, Wage$wage,col="gray")

fit.x1=lm(wage~bs(age,knots=c(25,40,60), degree=1),data=Wage)
preds1=predict(fit.x1,newdata=list(age=age.grid),se=T)
# lines(age.knots,preds$fit.x1,col="blue",lwd=2)

fit.x2=lm(wage~bs(age,knots=c(25,40,60), degree=2),data=Wage)
preds2=predict(fit.x2,newdata=list(age=age.grid),se=T)
# lines(age.knots,preds$fit.x2,col="red",lwd=2)

fit.x3=lm(wage~bs(age,knots=c(25,40,60), degree=3),data=Wage)
preds3=predict(fit.x3,newdata=list(age=age.grid),se=T)
# lines(age.knots,preds$fit.x3,col="green",lwd=2)

fit.ns=lm(wage~ns(age,df=4),data=Wage)
preds.ns=predict(fit.ns,newdata=list(age=age.grid),se=T)
# lines(age.knots,preds$fit.ns,col="black",lwd=2)
```